

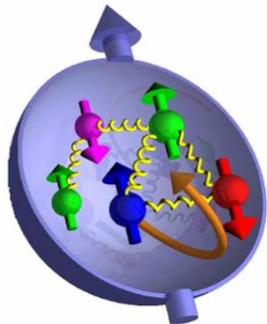
# Nucleon and nuclear polarized DIS structure functions

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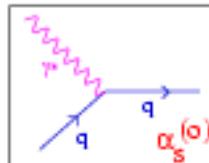


# Outlook

- Feynman Diagram
- Jacobi Model
- Data for analyzed
- Numerical Result
- Application for Jacobi
- Orbital Angular Momentum
- Conclusion

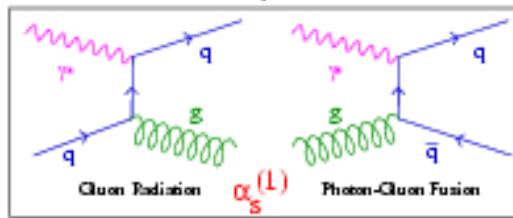
■ Diagram Feynman

**Beyond the Naive Parton Model**



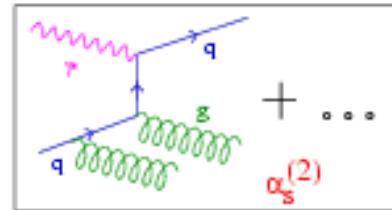
⇒ No gluons

$$g_1^0(x) = \frac{1}{2} \sum e_q^2 \Delta q(x)$$



⇒ Quarks are re-defined  
with the inclusion of  $\Delta G$   
(weak dependence)

$$g_1^{\text{LO}}(x, Q^2) = \frac{1}{2} \sum e_q^2 \Delta q(x, Q^2)$$



⇒  $g_1$  becomes  
explicitly  $\Delta G$  dependent

$$g_1^{\text{NLO}}(x, Q^2) = g_1^{\text{LO}} + \frac{\alpha_s}{2\pi} \frac{1}{2} \sum e_q^2 [\Delta q(x, Q^2) \otimes C_q + \Delta G(x, Q^2) \otimes C_G]$$

$$\delta q_{NS\pm}^n(Q^2) = \left\{ 1 + \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{2\pi} \left( -\frac{2}{\beta_0} \right) \left( \delta P_{NS\pm}^{(1)n} - \frac{\beta_1}{2\beta_0} \delta P_{qq}^{(0)n} \right) \right\} \\ \times L^{-(2/\beta_0)\delta P_{qq}^{(0)n}} \delta q_{NS\pm}^n(Q_0) + \mathcal{O}(\alpha_s^2) \quad \text{Non-Singlet}$$

$$\begin{pmatrix} \delta \Sigma^n(Q^2) \\ \delta g^n(Q^2) \end{pmatrix} = \left\{ L^{-(2/\beta_0)\delta \hat{P}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi} \hat{U} L^{-(2/\beta_0)\delta \hat{P}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi} L^{-(2/\beta_0)\delta \hat{P}^{(0)n}} \hat{U} \right\} \\ \times \begin{pmatrix} \delta \Sigma^n(Q_0^2) \\ \delta g^n(Q_0^2) \end{pmatrix} + O(\alpha_s^2), \quad \text{Singlet}$$

$$\hat{U} = -\frac{2}{\beta_0} (\hat{P}_+ \hat{R} \hat{P}_+ + \hat{P}_- \hat{R} \hat{P}_-) + \frac{\hat{P}_- \hat{R} \hat{P}_+}{\lambda_+^n - \lambda_-^n - \frac{1}{2}\beta_0} + \frac{\hat{P}_+ \hat{R} \hat{P}_-}{\lambda_-^n - \lambda_+^n - \frac{1}{2}\beta_0}$$

$$\hat{R} = \delta \hat{P}^{(1)n} - (\beta_1/2\beta_0) \delta \hat{P}^{(0)n} \quad \delta \hat{P}^{(1)}(x) = \begin{pmatrix} \delta P_{qq}^{(1)} & 2f \delta P_{qg}^{(1)} \\ \delta P_{gq}^{(1)} & \delta P_{gg}^{(1)} \end{pmatrix}$$

$$\hat{P}_\pm \equiv \pm \frac{\delta \hat{P}^{(0)n} - \lambda_\mp^n \mathbf{1}}{\lambda_+^n - \lambda_-^n}$$

$$\alpha_s(Q^2) \cong \frac{1}{b \log \frac{Q^2}{\Lambda_{MS}^2}} - \frac{b'}{b^3} \frac{\ln \left( \ln \frac{Q^2}{\Lambda_{MS}^2} \right)}{\left( \ln \frac{Q^2}{\Lambda_{MS}^2} \right)^2}, \quad b = \frac{33-2f}{12\pi} \\ b' = \frac{153-19f}{24\pi^2}$$

## ▪ Jacobi Model

$$g_1^p(N, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \left(1 + \frac{\alpha_s}{2\pi} \Delta C_q^N\right) [\Delta q(N, Q^2) + \Delta \bar{q}(N, Q^2)] + \frac{\alpha_s}{2\pi} 2 \Delta C_g^N \Delta g(N, Q^2) \right\},$$

Polarized Structure Function in moment Space

$$\Delta C_q^N = \frac{4}{3} \left[ -S_2(N) + (S_1(N))^2 + \left( \frac{3}{2} - \frac{1}{N(N+1)} \right) S_1(N) + \frac{1}{N^2} + \frac{1}{2N} + \frac{1}{N+1} - \frac{9}{2} \right],$$

$$\Delta C_g^N = \frac{1}{2} \left[ -\frac{N-1}{N(N+1)} (S_1(N)+1) - \frac{1}{N^2} + \frac{2}{N(N+1)} \right],$$

$$x g_1^{N_{max}}(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} \Theta_n^{\alpha, \beta}(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) M[x g_1, j+2],$$

Polarized structure Function in x space by Jacobi Method

$$\Theta_n^{\alpha, \beta}(x) = \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) x \quad \int_0^1 dx \ x^\beta (1-x)^\alpha \Theta_k^{\alpha, \beta}(x) \Theta_l^{\alpha, \beta}(x) = \delta_{k,l},$$

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$$x\delta u_v = A_{uv} \eta_{uv} x^{a_{uv}} (1-x)^{b_{uv}} (1+c_{uv}x)$$

$$x\delta d_v = A_{dv} \eta_{dv} x^{a_{dv}} (1-x)^{b_{dv}} (1+c_{dv}x)$$

$$x\delta \bar{q} = A_s \eta_s x^{a_s} (1-x)^{b_s}$$

$$x\delta g = A_g \eta_g x^{a_g} (1-x)^{b_g}$$

### Parton Distribution in $Q_0^2=4 \text{ GeV}^2$

$$A_i^{-1} = \left( 1 + c_i \frac{a_i}{a_i + b_i + 1} \right) B(a_i, b_i + 1)$$

$$a_3 = \int_0^1 dx \delta q_3 = F + D = 1.2670 \pm 0.0035$$

$$a_8 = \int_0^1 dx \delta q_8 = 3F - D = 0.585 \pm 0.025$$

$$a_3 = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d}),$$

$$a_8 = (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) - 2(\Delta s + \Delta \bar{s}) ,$$

$$\mathbf{M}[\delta f_i(x, Q_0^2)](N) = \int_0^1 x^{n-2} x \delta f_i(x, Q_0^2) dx \quad \text{Parton Distribution in Moment Space}$$

$$= \eta_i A_i \left( 1 + c_i \frac{N-1+a_i}{N+a_i+b_i} \right) B(N-1+a_i, b_i+1)$$

## ■ Data for analyzed

Experiment	x-range	Q <sup>2</sup> -range[GeV <sup>2</sup> ]	number of data points
E143(p)	0.031-0.749	1.27-9.52	28
HERMES(p)	0.023-0.66	1.01-7.36	19
HERMES(p)	0.23-0.66	2.5(Fixed)	20
SMC(p)	0.005-0.48	1.30-58.0	12
EMC(p)	0.015-0.466	3.50-29.5	10
E155	0.015-0.750	1.22-34.72	24
HERMES06(P)	0.0264-0.7311	1.12-14.29	66
COMPAS10(P)	0.0046-0.568	1.1-62.1	15
Proton			194
E143(d)	0.027-0.749	1.17-9.52	28
E155(d)	0.015-0.750	1.22-34.79	24
SMC(d)	0.005-0.479	1.30-54.8	12
HERMES06(d)	0.0328-0.7248	1.15-12.21	29
Deuteron			93
E142(n)	0.035-0.466	1.10-5.50	8
HERMES(n)	0.033-0.464	1.22-5.25	9
E154(n)	0.017-0.564	1.20-15	11
HERMES06(n)	0.0264-0.7311	1.12-14.29	37
Neutron			65
total			352

$$\chi^2_{\text{global}} = \sum_n w_n \chi^2_n, \quad (n \text{ labels the different experiments})$$

$$\chi^2_n = \left( \frac{1 - \mathcal{N}_n}{\Delta \mathcal{N}_n} \right)^2 + \sum_i \left( \frac{\mathcal{N}_n g_{1,i}^{\text{data}} - g_{1,i}^{\text{theor}}}{\mathcal{N}_n \Delta g_{1,i}^{\text{data}}} \right)^2.$$

CERN Program Library Long Writeup D506

# MINUIT



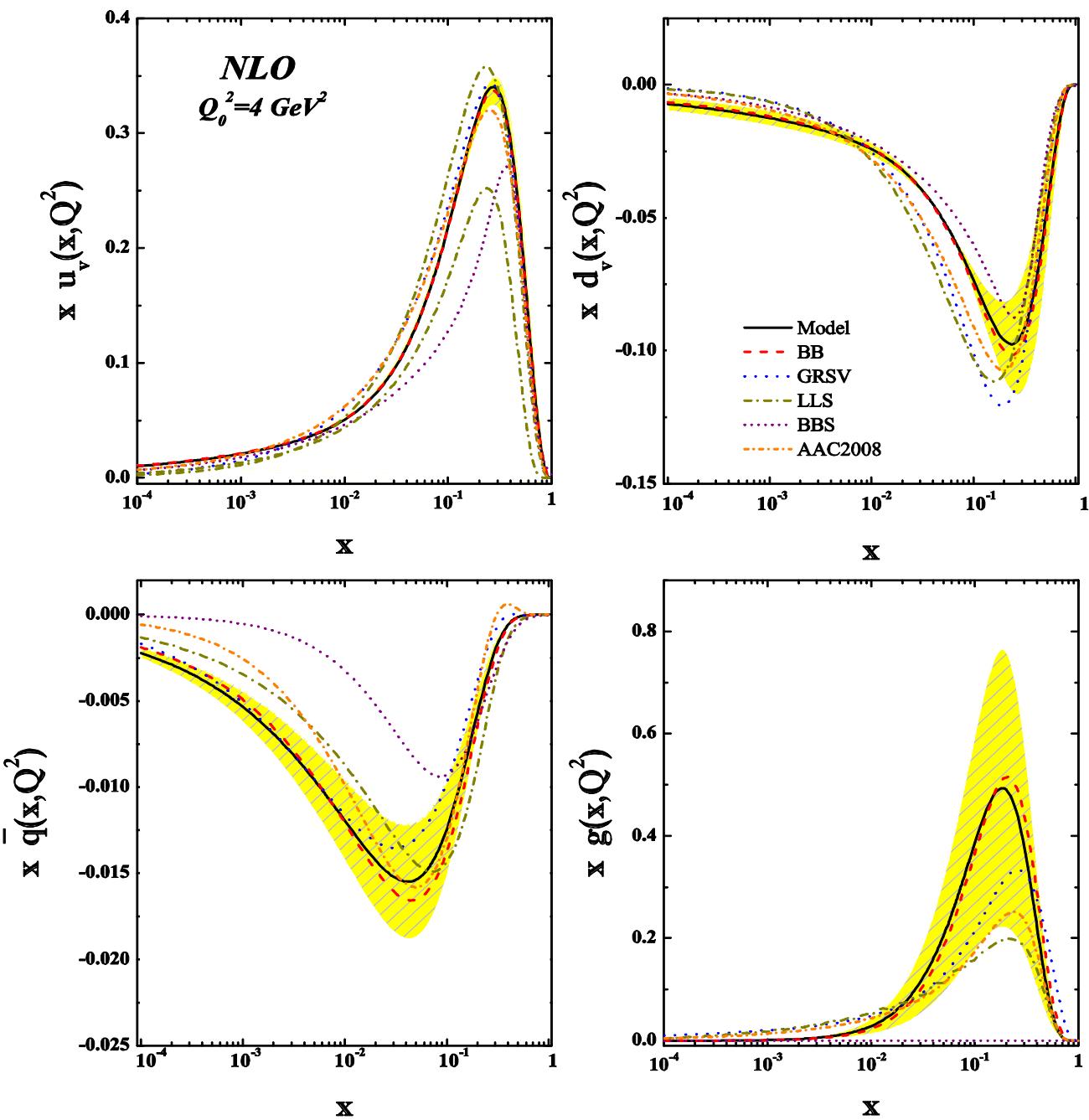
Function Minimization and Error Analysis

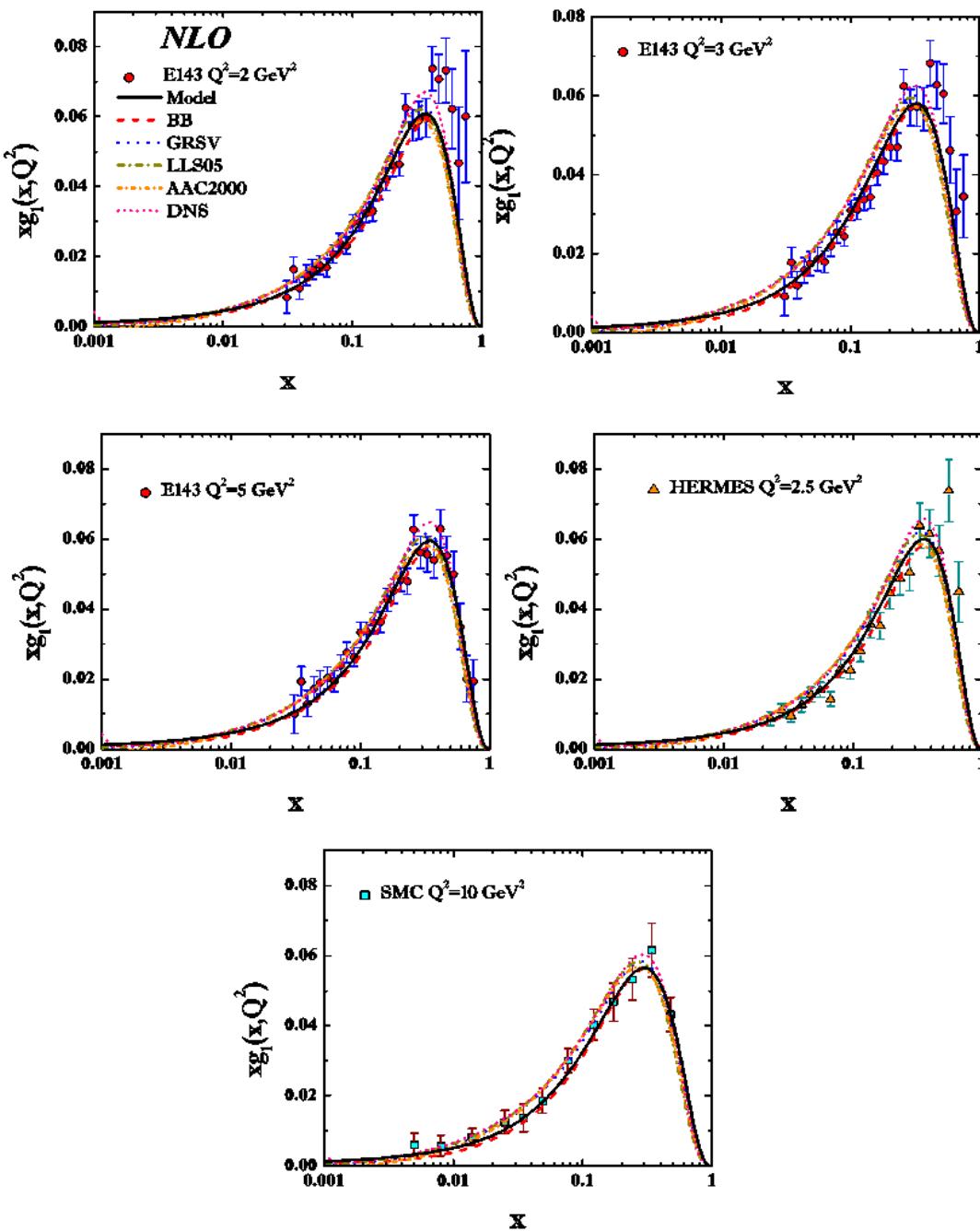
## ▪ Numerical Result

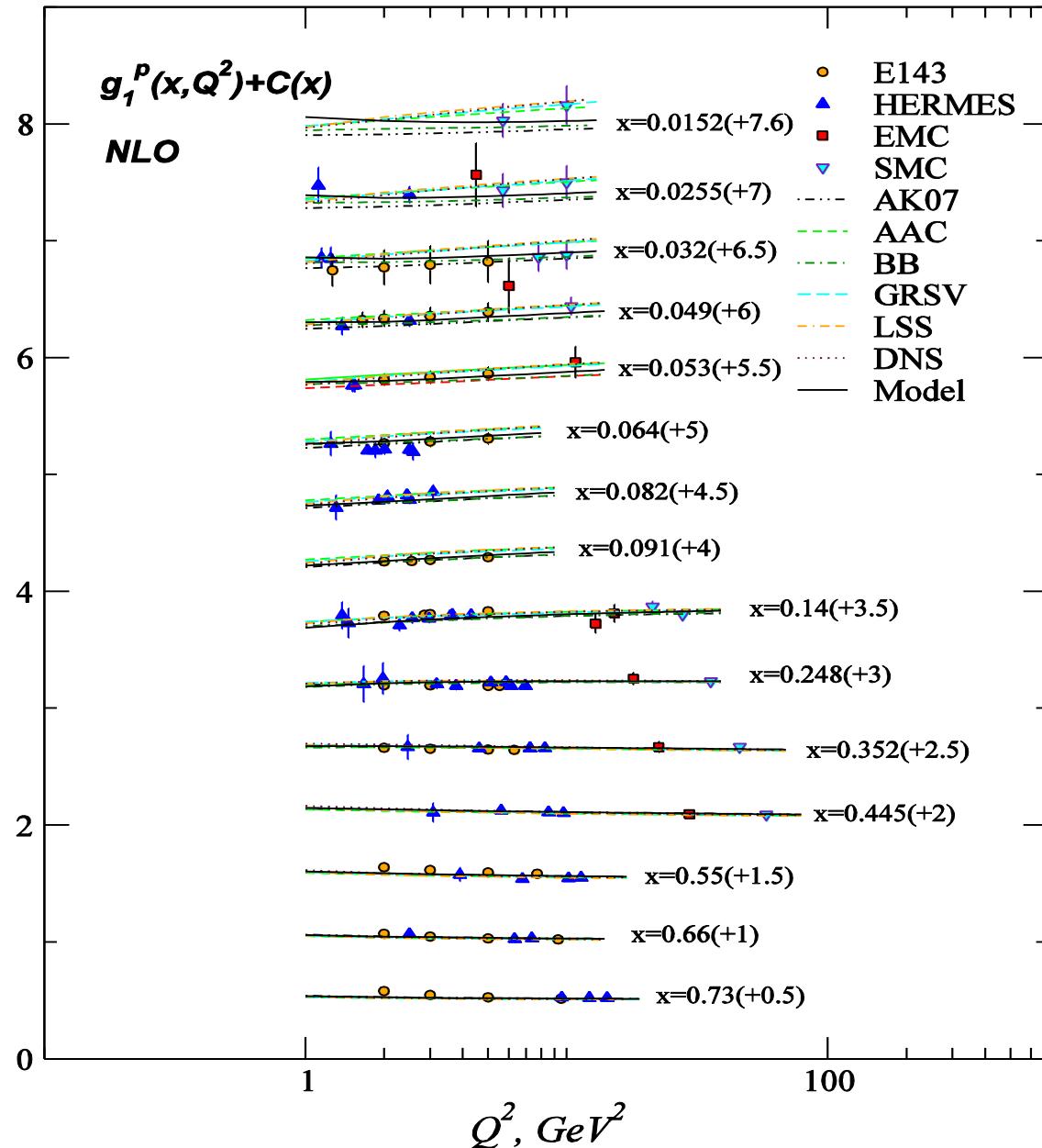
NLO									
	$\Lambda_{QCD}^{(4)}$	$a_{u_v}$	$b_{u_v}$	$a_{d_v}$	$b_{d_v}$	$\eta_{sea}$	$a_{sea}$	$\eta_G$	
$\Lambda_{QCD}^{(4)}$	2.08E-5								
$a_{u_v}$	2.08E-5	2.06E-5							
$b_{u_v}$	8.67E-4	3.67E-2	3.67E-2						
$a_{d_v}$	-2.46E-4	1.15E-4	-1.03E-2	3.04E-3					
$b_{d_v}$	2.00E-3	-9.08E-4	1.15E-4	-2.36E-2	2.04E-1				
$\eta_{sea}$	-6.49E-5	2.74E-5	-2.73E-3	-2.99E-2	-6.22E-3	2.05E-4			
$a_{sea}$	3.72E-5	-2.62E-5	2.33E-3	7.71E-4	5.65E-3	-1.73E-4	2.08E-4		
$\eta_G$	9.24E-5	1.05E-1	1.02E-2	-2.99E-2	2.46E-1	-7.89E-3	6.99E-3	3.09E-1	

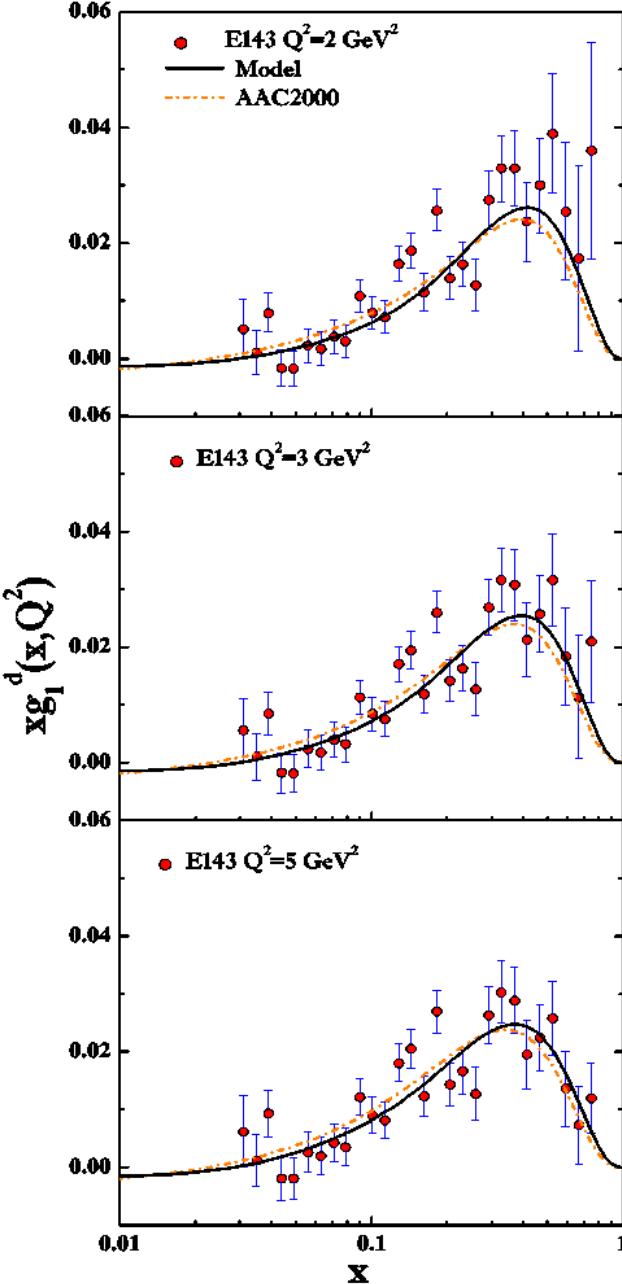
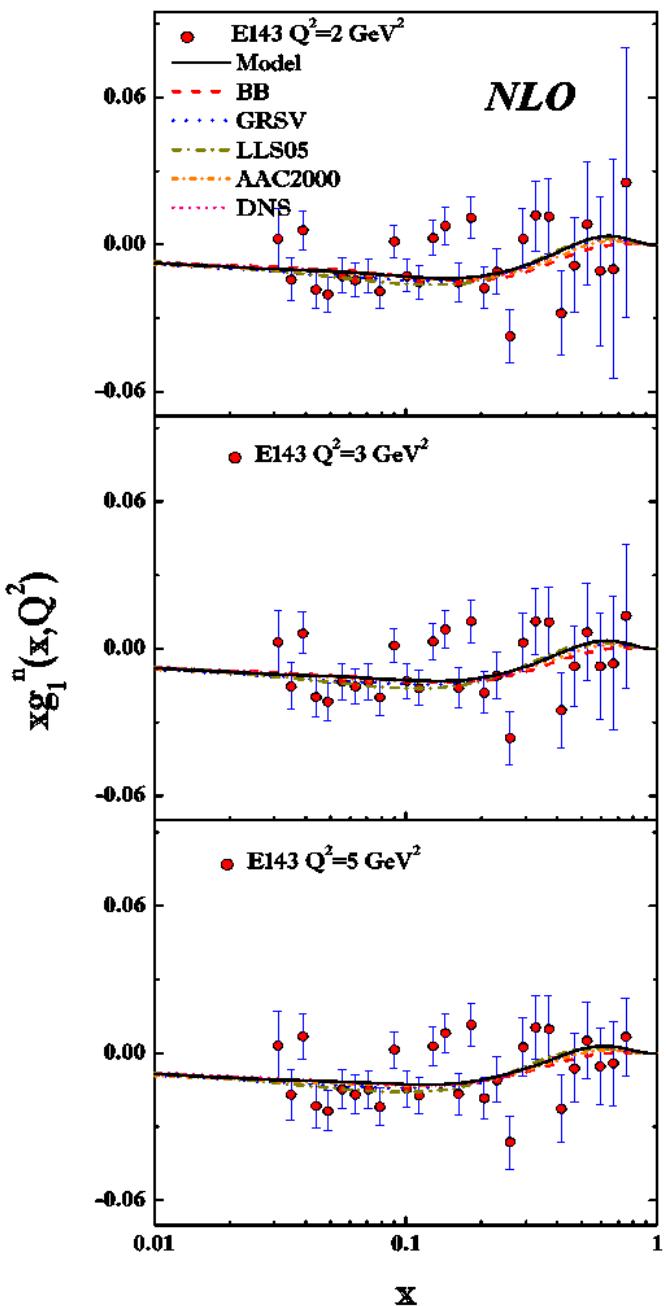
Covariance Matrix

	NLO	
	Value	Error
$\eta_{u_v}$	0.926	fixed
$a_{u_v}$	0.300	0.00454
$b_{u_v}$	3.17	0.191
$c_{u_v}$	27.1	fixed
$\eta_{d_v}$	-0.341	fixed
$a_{d_v}$	0.235	0.0552
$b_{d_v}$	3.412	0.452
$c_{d_v}$	19.05	fixed
$\eta_s$	-0.0743	0.0143
$a_s$	0.384	0.0144
$b_s$	6.178	fixed
$\eta_g$	1.008	0.0143
$a_g$	1.39	fixed
$b_g$	6.178	fixed
$\Lambda_{QCD}^{(4)}, MeV$	215	45.6
$\chi^2/NDF$	$333.4/344 = 0.969$	

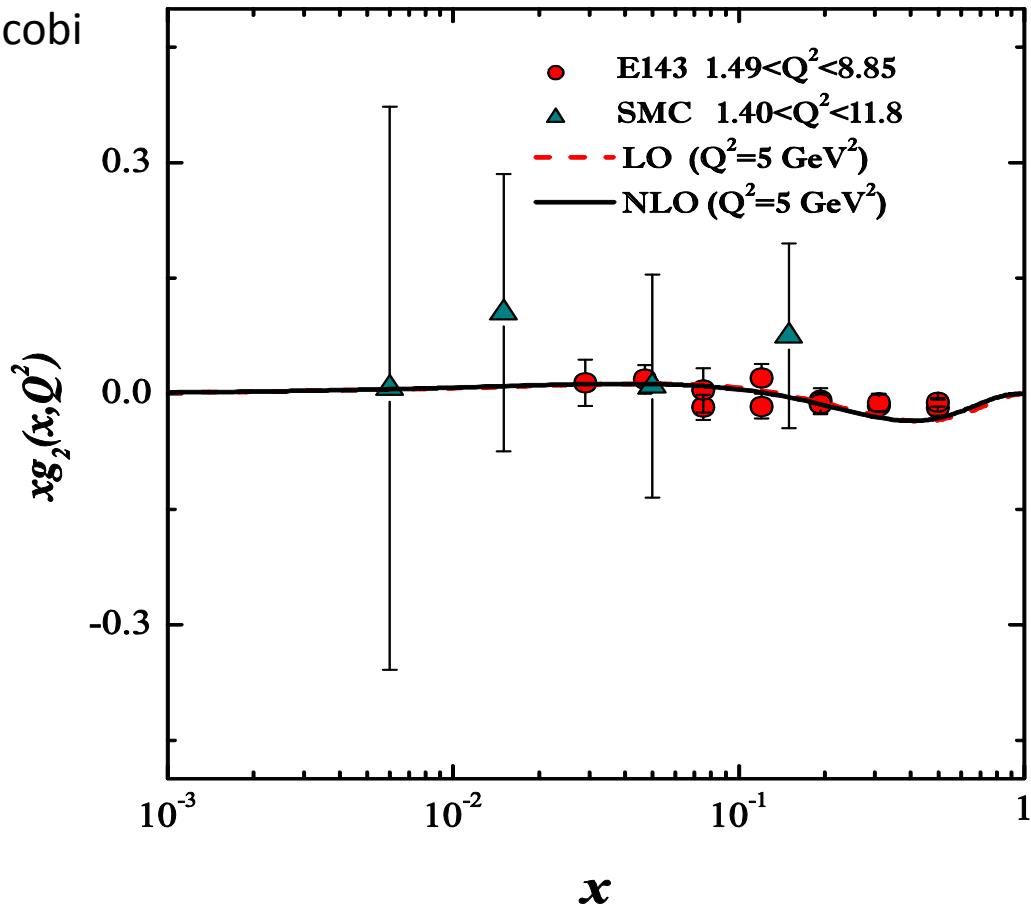








■ Application for Jacobi



$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy$$

$$g_1^{^3He}(x, Q^2) = \int_x^3 \frac{dy}{y} \Delta f_{^3He}^n(y) g_1^n\left(\frac{x}{y}, Q^2\right) + 2 \int_x^3 \frac{dy}{y} \Delta f_{^3He}^p(y) g_1^p\left(\frac{x}{y}, Q^2\right)$$

$$g_1^{^3H}(x, Q^2) = 2 \int_x^3 \frac{dy}{y} \Delta f_{^3H}^n(y) g_1^n\left(\frac{x}{y}, Q^2\right) + \int_x^3 \frac{dy}{y} \Delta f_{^3H}^p(y) g_1^p\left(\frac{x}{y}, Q^2\right)$$

$$\Delta f_{^3He}^n(y) = \frac{a^n e^{-\frac{0.5(1-d^n)(-b^n+y)^2}{c^{n2}}}}{1 + \frac{d^n(-b^n+y)^2}{c^{n2}}}$$

$$\Delta f_{^3He}^p(y) = \frac{\sum_{i=0}^4 a_i^p U_i(y)}{\sum_{i=0}^4 b_i^p U_i(y)}$$

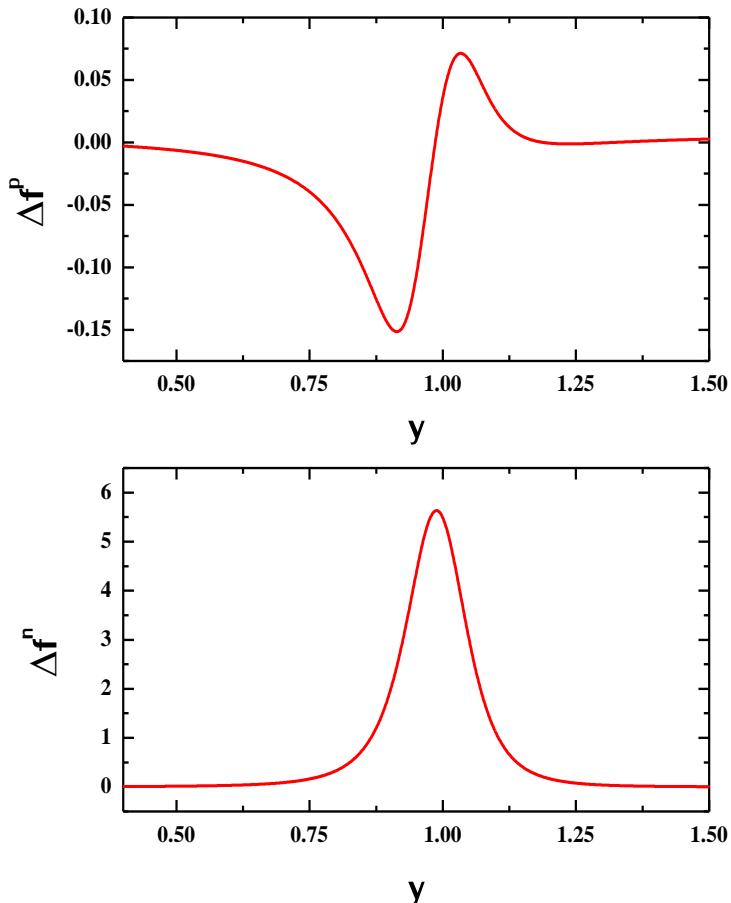
$$a_n = 5.6505568 \quad a_0^p = 0.0148376 \quad b_0^p = 4.15388$$

$$b_n = 0.9868182 \quad a_1^p = -0.0189575 \quad b_1^p = -4.75525$$

$$c_n = 0.0644468 \quad a_2^p = 0.0121792 \quad b_2^p = 2.68417$$

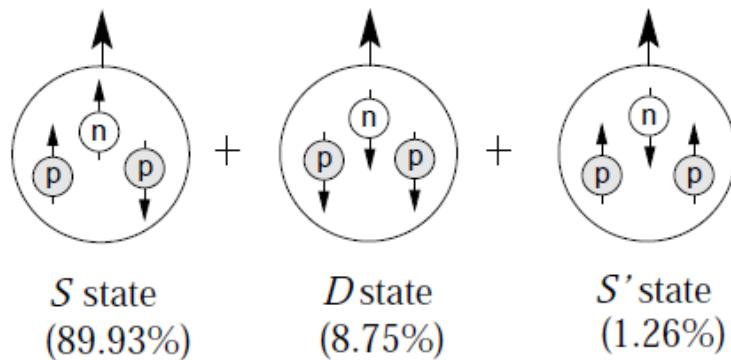
$$d_n = 0.8076502 \quad a_3^p = -0.0040397 \quad b_3^p = -0.800306$$

$$a_4^p = 0.0005408 \quad b_4^p = 0.101095$$



$$g_1^{^3\text{He}} = \int_x^3 \frac{dy}{y} \Delta f_{n/^3\text{He}}(y) \tilde{g}_1^n(x/y) + \int_x^3 \frac{dy}{y} \Delta f_{p/^3\text{He}}(y) \tilde{g}_1^p(x/y)$$

$$- 0.014 (\tilde{g}_1^p(x) - 4\tilde{g}_1^n(x)) + a(x) g_1^n(x) + b(x) g_1^p(x),$$

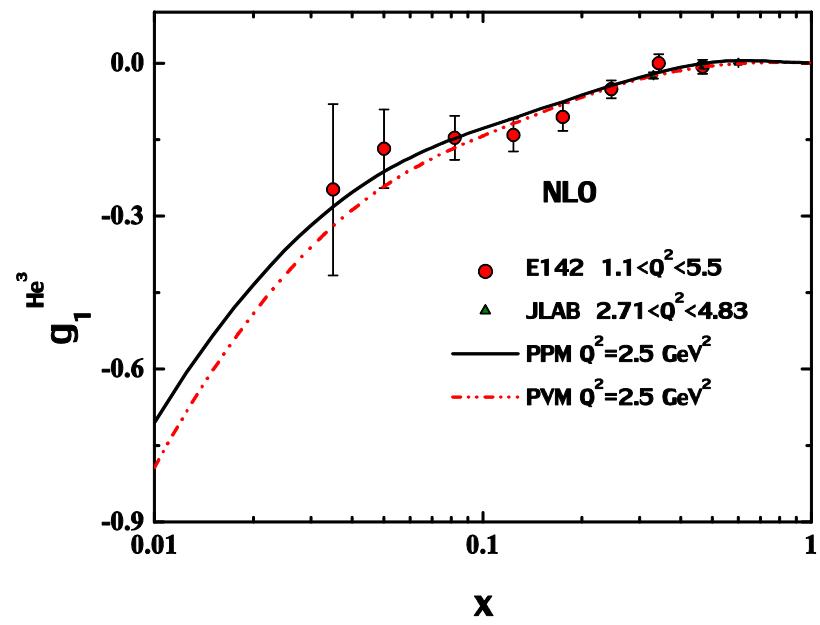
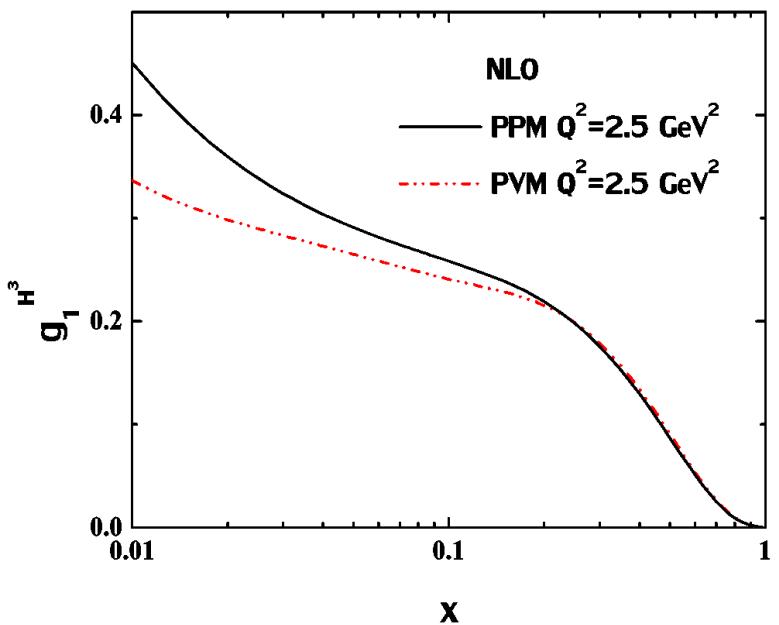


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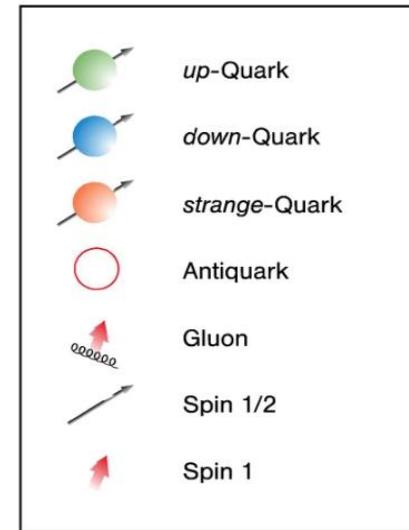
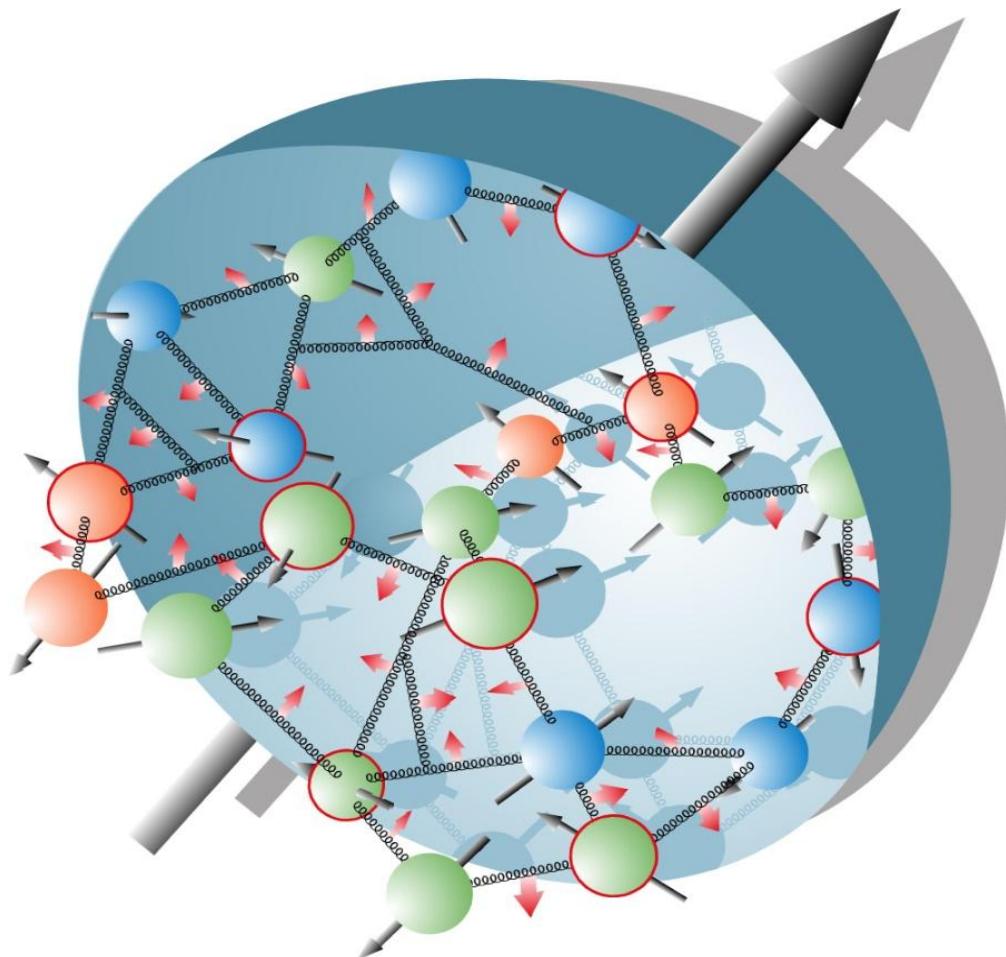
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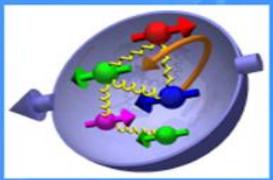


- Orbital Angular Momentum



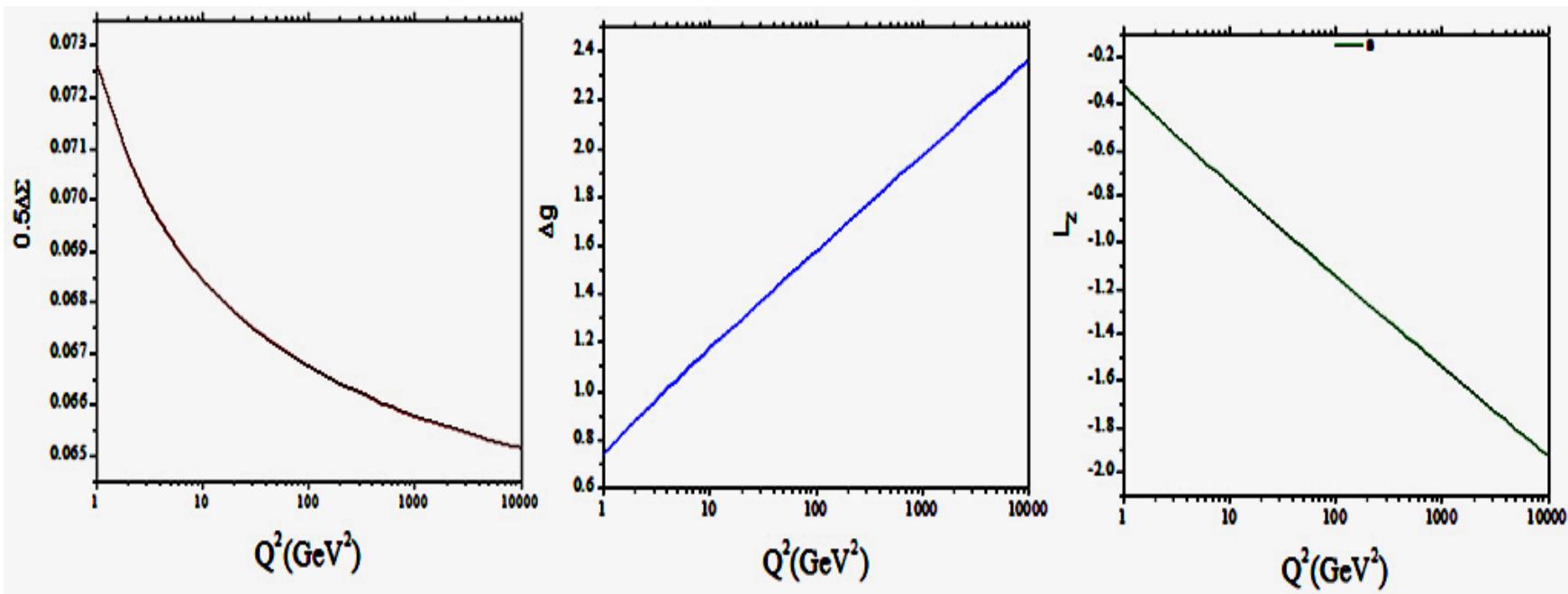
Distribution	Model(NLO)	BB(Iset3)	GRSV	AAC
$Q^2 = 4 \text{ GeV}^2$	$\Delta u_v$	0.926	$0.926 \pm 0.071$	0.926
	$\Delta d_v$	-0.341	$-0.341 \pm 0.123$	-0.3409
	$\Delta \bar{q}$	-0.0743	$-0.074 \pm 0.017$	-0.0625
	$\Delta G$	1.008	$1.026 \pm 0.549$	0.6828
First Moment	$Q^2 = 2 \text{ GeV}^2$	$Q^2 = 3 \text{ GeV}^2$	$Q^2 = 5 \text{ GeV}^2$	$Q^2 = 10 \text{ GeV}^2$
$\Delta u_v$	0.9278	0.9266	0.9255	0.9243
$\Delta d_v$	-0.3416	-0.3412	-0.3408	-0.34038
$\Delta \bar{q}$	-0.074080	-0.0741803	-0.0743	-0.0744691
$\Delta G$	0.8786	0.9554	1.049	1.174
$\Gamma_1^p$	0.1248	0.1256	0.1265	0.1274
$\Gamma_1^n$	-0.06681	-0.06748	-0.06816	-0.06886
$\Gamma_1^d$	0.02647	0.02656	0.0266	0.02674

**Table 4:** The first moments of polarized parton distributions,  $\Delta u_v$ ,  $\Delta d_v$ ,  $\Delta \bar{q}$ ,  $\Delta g$  and  $\Gamma_1^p$  in the NLO approximation for some value of  $Q^2$ .



Full description of  $J_q$  &  $J_g$   
needs  
orbital angular momentum

$$\frac{1}{2} = \frac{1}{2} \underbrace{(\Delta u_v + \Delta d_v + \Delta q_s)}_{\Delta \Sigma} + \Delta G + L_q + L_g$$



# Conclusion

- Jacobi Model good agreement with Experimental data
- By Jacobi Model we can determined  $xg_2$  and good agreement with experimental data
- By Jacobi Model we can determined  $g_1(x, Q^2)$  for  $\text{He}^3$  and  $\text{H}^3$
- By Jacobi Model we can determined TMC (Target Mass Correction)
- We analyzed with recently data from Compass 2010
- Study about spin of proton and find behavior of orbital angular momentum of quark & gluon
- Compare with another group and get good result about parton distribution

# Thank You

